EFFECT OF AN IMPERMEABLE INCLUSION IN THE UNDERLYING HIGHLY PERMEABLE PRESSURIZED HORIZON ON THE CONDITIONS OF GROUND WATER IN AN IRRIGATED LAYER OF SOIL

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The problem of plane, nonpressurized, steady-state filtration through a layer of soil into an underlying pressurized horizon, which contains an impermeable section at the top, with uniform infiltration on the free surface is solved in a hydrodynamic formation. A constructive solution of the problem is given with the help of the method of P. Ya. Polubarinova-Kochina; representations are obtained for the characteristic dimensions of the flow scheme and the depression. The case of limiting flow — no head in the bottom, highly permeable layer — studied in [1] is noted.

The flow scheme under study is shown in Fig. 1. Infiltration water, seeping through a layer of soil into an underlying highly permeable layer, the head at the top of which is constant and equal to H_0, forms a mound of groundwater above the impermeable section, which is modeled by a segment of length 2L, and spreads laterally to infinity. It is assumed that the intensity of the infiltration (referred to the coefficient of filtration of the soil) is constant and equals ε.

The y axis, being a line of symmetry, is a streamline. We shall examine the right half of the region of the flow. The study of the model described reduces to determining the depression curve AB, bounding the region z, and two mutually conjugate, in this region, functions φ and ψ with the boundary conditions

$$\begin{gathered}(\varphi + y)|_{AB} = H_0, \ \varphi|_{BC} = 0, \\ (\psi - \varepsilon x)|_{AB} = \psi|_{AD} = \psi|_{CD} = 0,\end{gathered} \tag{1}$$

where φ and ψ are the velocity potential and the stream function, respectively, referred to the filtration coefficient of the soil.

The problem is solved by the method in [2], which is based on the use of the analytic theory of linear differential equations. The canonical region considered here is the rectangle in the plane $\tau = \tau_1 + i\tau_2$ (Fig. 2), where $\rho = K'/K$ (K(k) is the complete elliptic integral of the first kind with modulus k, $K' = K(k')$, $k' = \sqrt{1 - k^2}$).

We introduce the functions $z(\tau)$ and $\omega(\tau)$ which conformally map the indicated rectangle into the region of filtration $z = x + iy$ and the region of the complex potential $\omega = \varphi + i\psi$, as well as the functions

$$Z = dz/d\tau, \ \Omega = d\omega/d\tau, \tag{2}$$

which must be determined.

The function which gives the conformal mapping of the rectangle on the region of the complex velocity $w = d\omega/dz$ (Fig. 3), corresponding to the boundary conditions (1), is written in the form [3]

$$w(\tau) = \sqrt{\varepsilon}\, i\, \frac{\vartheta_2(\tau + \alpha i) - \vartheta_2(\tau - \alpha i)}{\vartheta_2(\tau + \alpha i) + \vartheta_2(\tau - \alpha i)}, \tag{3}$$

where $\vartheta_2(\tau)$ is the second Jacobi ϑ-function with the parameter $q = \exp(-\pi\rho)$; $\alpha = (1/2\pi) \ln[(1 + \sqrt{\varepsilon})/(1 - \sqrt{\varepsilon})]$.

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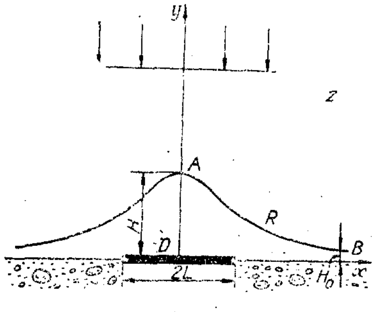

Fig. 1

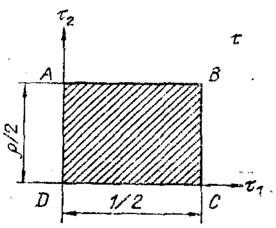

Fig. 2

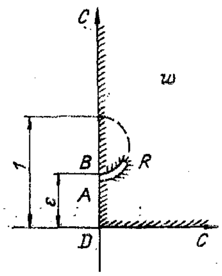

Fig. 3

TABLE 1

H_0/L	ε=0,2		ε=0,5		ε=0,8	
	k	H/L	k	H/L	k	H/L
0	0,9997184	0,5000	0,9708201	1,0000	0,769110	2,0000
0,001	0,9997144	0,5002	0,9707375	1,0003	0,7680781	2,0003
0,1	0,9991045	0,5274	0,9616661	1,0318	0,7553391	2,0351
0,2	0,9975367	0,5695	0,9505844	1,0697	0,7413497	2,0727
0,4	0,9882379	0,6863	0,9226084	1,1619	0,7129412	2,1548
0,6	0,9660830	0,8313	0,8877010	1,2726	0,6842856	2,2459
0,8	0,9302154	0,9928	0,8476825	1,3983	0,6557589	2,3455
1,0	0,8843398	1,1645	0,8047256	1,5361	0,6276766	2,4529
10,0	0,1555625	10,0196	0,1548670	10,0779	0,1521975	10,3065

Taking into account the behavior of the functions (2) near singular points, and also the expression (3), we obtain a parametric solution of the starting boundary-value problem:

$$Z = C\frac{\vartheta_2(\tau+\alpha i)+\vartheta_2(\tau-\alpha i)}{\vartheta_3(\tau)},\quad C>0; \tag{4}$$

$$\Omega = C\sqrt{\varepsilon}\, i\,\frac{\vartheta_2(\tau+\alpha i)-\vartheta_2(\tau-\alpha i)}{\vartheta_3(\tau)}. \tag{5}$$

The validity of the formulas (3)-(5) is established by a direct check.

Writing the representation (4) for different sections of the boundary of the region τ followed by integration gives parametric equations for the corresponding boundary sections of the scheme.

We note the limiting case $H_0 = 0$ (no head), associated with the degeneracy of the region of the complex velocity. For $\alpha = 0.5\rho$, $w = \sqrt{\varepsilon}\ \mathrm{tg}\ \pi\tau$, $Z = C\cos\pi\tau$, $\Omega = C\sqrt{\varepsilon}\sin\pi\tau$. The semicircle $|w - 0.5(1+\varepsilon)i| < 0.5(1-\varepsilon)$, drops out of the region w, and the depression curve emerges onto the top of the underlying layer at a right angle at some point B, which coincides with the point of the inflection R, so that the entire depression curve becomes convex. For this scheme, studied in [1], we have (L_1 is the abscissa of the point B)

$$L_1/L = \sqrt{1/(1-\varepsilon)},\quad H/L = \sqrt{\varepsilon/(1-\varepsilon)}. \tag{6}$$

In the direct physical formulation, the quantities L and H_0, obtained by integrating (4) from the point D to the point C and from the point D to the point B using the point A, are parameters of the mapping C and the modulus k. In the algorithm programmed for a computer k is found from the relation for H_0 by the halving method, and the value of H and

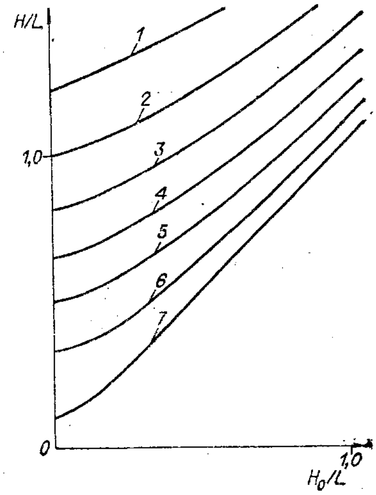

Fig. 4

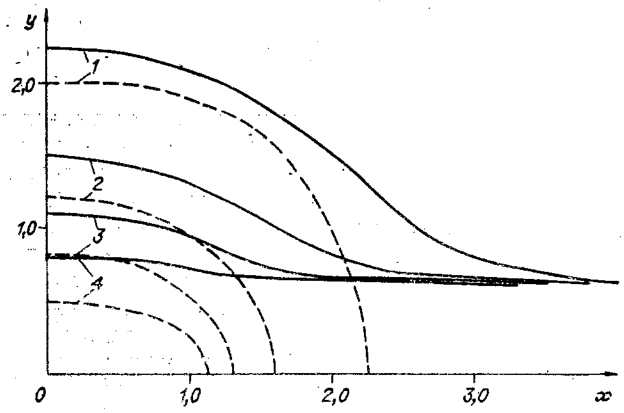

Fig. 5

also the coordinates of the points on the depression curve are calculated by first eliminating C from the relation fixing L = 1.

According to the calculations, with fixed ε the function $k = k(H_0/L)$ is a monotonically decreasing function and has an upper limit $k_* = k(0)$, which corresponds to the limiting case $H_0 = 0$ and the maximum admissible value for the flow scheme under study; the modulus k_* is determined from the equation

$$K'/K = (1/\pi)\ln[(1+\sqrt{\varepsilon})/(1-\sqrt{\varepsilon})]. \tag{7}$$

In addition, for $k \leq 0.707106$ (which corresponds to the case when $K \leq K'$) the ϑ-functions are expanded in a series in powers of the parameter q, and in the opposite case the expansions in $q' = \exp(-\pi\rho')$, where $\rho' = 1/\rho$, are used.

Table 1 shows the results of calculations of k and H/L as a function of H_0/L for some values of ε; the first row contains the values of k_* and H_*/L found from (7) and (6), respectively.

Figure 4 shows the dependence of H/L on ε and H_0/L (the lines 1-7 for ε = 0.6; 0.5; 0.4; 0.3; 0.2; 0.1; 0.01); for $H_0/L > 1$ it is nearly linear.

Figure 5 shows the depression curves calculated with $H_0/L = 0.6$ and different values of ε (the lines 1-4 for ε = 0.8; 0.6; 0.4; 0.2); for the same values of ε the broken lines correspond to the limiting case $H_0 = 0$.

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SELF-SIMILAR SOLUTIONS OF A SYSTEM OF TWO PARABOLIC EQUATIONS

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The description of many physical systems comes down to the solution of a system of two nonlinear equations of the parabolic type. Such systems can be the electron–hole plasma of a semiconductor and a weakly ionized gas plasma, nonequilibrium superconductors, as well as a number of chemical and biological objects, the properties of which are determined by autocatalytic reactions. The formation of complicated nonuniform structures occurs upon the loss of stability in these systems. We shall examine the concrete problem of the development of an ionization–superheating instability in a self-maintained discharge, described by the equation of charged-particle balance of the plasma and the equation of heat balance. The mechanism of this stability is connected with the decrease in the density of gas escaping at constant pressure from a superheated region, and with the rise in electron temperature occurring as a consequence of this (see, e.g., [1]). Self-similar functions for the local values of the charged-particle density and the gas temperature, being solutions of the corresponding balance equations, are of interest for the understanding of the nonlinear state of this process. An ionization–superheating instability in a high-frequency field and a self-similar solution, describing the explosive development of conductivity in a constricting discharge, neglecting the thermal conductivity of the gas and charge recombination, were studied in [2]. Self-similar solutions of a pair of equations of the parabolic type under the conditions of a self-maintained glow discharge are investigated in the present paper. The solutions obtained can be of interest for a whole series of physical systems.

Let an electric discharge be ignited between two electrodes spaced a distance L apart. Assuming that it is uniform along the current, we use balance equations for the charged-particle density n and the gas temperature T:

$$\partial n/\partial t - D_a \Delta n = \nu_i n - \beta n^2; \tag{1}$$

$$\frac{1}{T}\frac{\partial T}{\partial t} - \chi \frac{1}{T}\Delta T = \frac{\sigma E^2}{c_p p}. \tag{2}$$

Here D_a and χ are the coefficients of ambipolar diffusion and thermal diffusivity, respectively; ν_i is the frequency of ionization by electron impact; β is the dissociative-recombination constant; c_p is the reduced heat capacity of the gas; $\sigma = e^2 n/m\nu_m$ is the conductivity of the discharge plasma, which neglecting electron–electron collisions, is proportional to the electron density. In writing (1) and (2), it was assumed that the time of pressure equalization is small compared with the characteristic time of development of instability. This is possible if the pressure does not increase with time owing to the presence of a large ballast volume.

The ionization frequency is usually a sharply growing function of the parameter $E/N \sim ET$ (N is the gas density). Under the conditions of a gas discharge, the approximation $\nu_i = A \exp(-Bp/ET)$ is used for the frequencies [1], where A, B = const.

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